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Relationship between fracture spacing and bed thickness

F. L. LADEIRA

Alameda Prof. Fabio Ribeiro Gomes, 1/504 Vicosa, M.G. Brazil

and

N. J. PRICE

Department of Geology, Imperial College, Prince Consort Road,
London SW7 2BP, England

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Abstract—Empirical relationships between fracture spacing and bed thickness are established for a number of rock types. The influence of the thickness of adjacent incompetent layers and also the lithology of the competent beds on fracture spacing are indicated; and possible mechanisms leading to the development of these relationships are briefly discussed.

INTRODUCTION

THERE is no simple and universally accepted definition of the geological feature termed a 'joint'. A large number of the features studied here would certainly be termed joints by most field geologists. There were other features, however, about which discussion, as regards nomenclature, would most certainly arise. This follows because the various sets studied in the field comprise individual features of different ages. All the features studied were fractures, so we have opted to use the more omnibus term. The manner in which different sets and different ages of fractures within sets may develop has been discussed elsewhere (Price 1974).

It has long been established that the lithology and thickness of competent beds influence the spacing of fractures that cut and are contained by them. Thus, it was indicated by Bodgonov (1947), Novikova (1947) and Kirillova (1949) that the spacing (S) between fractures varied as:

$$S = K \,.\, B_t \qquad (1)$$

where B_t is the thickness of the bed and K is some constant which is related to the lithology of the bed.

Various mechanisms to explain the relationships have been proposed (e.g. Price 1966, Hobbs 1967, Sowers (1973), and all these analyses predict that there should be a linear relationship between fracture spacing and bed thickness. Consequently, despite a warning by Norris (1966) and the findings of Mastella (1972), the linearity of this relationship is still widely accepted.

However, even a casual inspection of the fracture spacing which is exhibited by very thick, massive competent beds (so well exposed in some desert or arid areas such as the buttes and narrow pinnacles of Monument Valley or the cliffs of the Grand Canyon, U.S.A.), where the fractures are relatively closely spaced, must immediately lead one to suspect the general validity of the simple linear relationship given by Equation (1). The reason for this apparent paradox mainly results from the fact that the early field investigations, on which the linear relationships are based, were conducted in sedimentary sequences in which bed thickness did not exceed 1.5 m. It was therefore decided to conduct further field measurements in sediments in which individual beds exceed 1.5 m in thickness. To this end, thousands of measurements were made (by F.L.L.) in the Carboniferous turbidites of the Alentejo area (Portugal), in the Carboniferous flysch of Devon and Cornwall (U.K.) and in the Jurassic limestones of Figueira da Foz (Portugal); so that the relationships between fracture spacing and bed thickness could be established for two markedly different lithological rock types.

Subsequent to the conclusion of these field studies, it came to the attention of the authors that a study of the development of fracture spacing in thick limestones had previously been conducted by McQuillan (1973). We were therefore able to use his data to augment that collected during this study.

Besides bed thickness and lithology, the other parameter known to influence fracture spacing is the degree of deformation experienced by the rocks. The influence of this parameter was demonstrated by Harris *et al.* (1960) from data collected around the Goose Egg Dome, Wyoming: a structure which has a wavelength in excess of 1.5 km. In the present study, measurements were made in sediments which exhibit a tectonic style and degree of deformation which is comparable in all the areas. Typically, the sediments were deformed to form folds with wavelengths of the order of 200 m. In profile, the limbs are generally planar, with the crests and troughs of the folds being sharply rounded (i.e. they are chevron-type folds). Measurements were taken throughout these structures and although many tens or even hundreds of measurements were taken in one bed around a fold, only the average fracture separation for a given bed is presented. Consequently, the data given here cannot be used to establish the influence of deformation. However, as we shall see, the possible influence of variations in degree of deformation is not vital to our study, for it in no way masks the fundamental control exhibited by bed thickness and lithology.

GRAPHICAL PRESENTATION OF DATA

One of the problems relating to the study of fracture separation is how the data may best be presented. In the majority of studies, the data are represented graphically with the ordinate and abscissa having linear scales so that the relationship expressed by equation (1) may readily be seen. This method has the advantage of simplicity; however, as we shall demonstrate, data relating to closely spaced fractures in thin beds result in an unresolvable 'blob' of information near the origin. Nevertheless the method makes clear certain relationships which are not obvious if other methods of presentation are employed.

A different method of presentation in which data are expressed as the number of fractures per unit length of traverse (here we take this to be 1.0 m) against bed thickness on a log–log plot, is shown in Fig. 1. This method of presenting data is similar to that used by McQuillan (1973), except that he used a linear/log plot, and obviates the previous problem relating to the unresolvable data near the origin. However, this mode of presentation makes it necessary to infer the actual fracture separation. In subsequent figures we will use the traditional method of presenting data. The type of presentation shown in Fig. 1 has been used so that the data from thin beds may readily be represented and the influence of a hitherto unsuspected parameter revealed.

THE INFLUENCE OF ADJACENT INCOMPETENT BEDS

As in previous studies, the present investigation was primarily concerned with the fracture separation in the competent units of a sedimentary sequence. However, as the present study progressed it became apparent that the development of fractures in the competent beds was also related to the thickness of the adjacent layers of incompetent material. For the sediments studied, the critical thickness of incompetent material appeared to fall within the range 4–6 cm. Consequently, we arbitrarily chose the critical thickness of 5 cm (see Fig. 1). As may clearly be seen in this figure, the number of fractures per metre in the competent layers which adjoin incompetent layers thicker than 5 cm is significantly smaller (for a given thickness of competent layer) than when the adjacent layers are thinner than 5 cm. Or, expressed in terms of fracture separation, when adjacent layers are 'relatively thick' the fractures in the competent layers are a little more widely spaced than when the adjoining incompetent layers are thin.

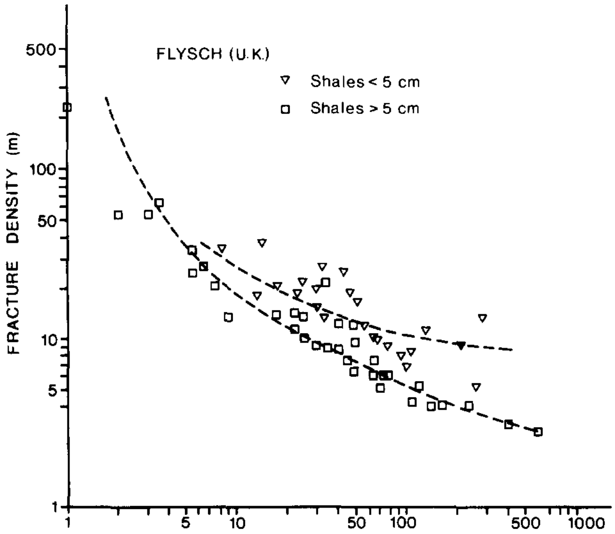

Fig. 1. Relationship between bed thickness and fracture frequency for different thickness of adjacent incompetent layer.

The influence of the thickness of the incompetent layer is less obvious if the data are plotted with linear scales on the ordinate and abscissa (see Fig. 2 which also includes data obtained from greywackes and limestones from Portugal). However, in order to differentiate more clearly between the data from beds of different lithologies and between data obtained from beds with 'thick' and thin adjacent incompetent layers, it was necessary to exaggerate (by a factor of five) the scale of the ordinate relative to the abscissa. Even with this horizontal exaggeration there is a confusion of data points near the origin. Moreover, the exaggeration has made more obvious possible departures from linearity of the relationships for relatively thin beds and has diminished the importance of the relationship between fracture separation and thickness for the thick beds. Nevertheless, it has not completely masked the fact that for the greywackes the fracture separation is approximately constant for bed thickness of greater than 1.0 m for the U.K. greywackes and greater than about 2.0 m for the Portuguese greywackes.

The curves A, B and C in Fig. 2 are again represented in Fig. 3(a) as curves a, b and c. In this latter figure the scale of the ordinate and abscissa are the same; and the relationships between bed thickness and fracture separation are more obvious than in Fig. 2. In addition, a curve based on data obtained by McQuillan (1973) is also presented in Fig. 3(a). The data published by McQuillan (1973) relate to measurements taken in beds of the Asmari Limestone in a number of major folds forming part of the Zagros ranges. The data fell into six main groups as far as bed thickness is concerned. These groups are clearly shown in Fig. 3(a); the range of thicknesses for the groups are indicated by error bars. The average fracture separation for each of these groups is shown by the small solid circle. This average is based on a number of sets of observations [the number of sets are indicated within brackets in Fig. 3(a)] while the range of values of these sets is also indicated by error bars.

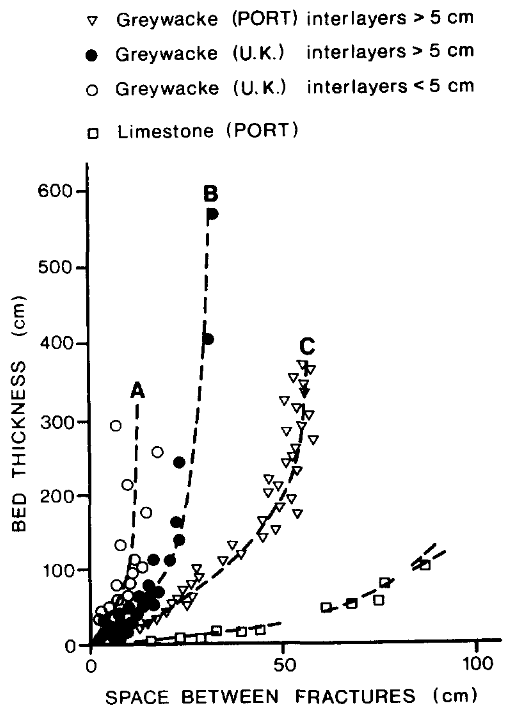

Fig. 2. Relationship between fracture separation and bed thickness for different rock types. Each point represents the mean of at least 50 readings.

The data presented by McQuillan (1973) were obtained from widely spaced localities in different folds. Hence, variations in lithology and the degree of tectonic deformation will have influenced the fracture separation. Nevertheless, despite these masking effects one can conclude that, as with the greywackes, the mean fracture separation in thick limestone beds is sensibly independent of bed thickness (provided individual units exceed 1.5 m thick).

INTERPRETATION OF DATA

We suggest that these various data can be represented by two straight line relationships [see Fig. 3(b)] which are the result of two different mechanisms determining fracture development.

The first of these relationships, indicated by line OA in Fig. 3(b) is the linear relationship forecast by the various theoretical treatments already cited, in which fracture separation is influenced by traction at the competent–incompetent interface, and will not be commented on further.

The second relationship for which fracture separation in beds of a given lithology is independent of bed thickness, is represented by line BC in Fig. 3(b). This relationship, we suggest, results from the hydraulic fracture mechanism. This mechanism will operate or be important in the development of any extension fracture or hybrid extension/shear fracture which develops in sediments.

As far as the writers are aware, the proportion of extension and hybrid fractures which develop in thick sedimentary units, relative to shear fractures, has not been quantified. However, from studies of fracture traces, rose diagrams, etc. (see e.g. Norman *et al.* 1977, Hancock & Kadhi, 1978) we suggest that extension and hybrid fractures are commonly dominant. In any event, they certainly constitute a very important proportion of the total number of fractures in thick layers. With few exceptions, total stresses in the crust are compressive. Extension fractures develop when the effective stress is tensile. This situation exists when the fluid pressure is higher than the least total stress. Consequently, the hydraulic fracture mechanism will play either an important or a dominant role in the development of extension fractures in thick units. Unless the fractures develop above the water table, water will fill the void spaces in the rock. Hence a fluid pressure, however small, will occur in the rock and consequently the hydraulic fracture mechanism will contribute to the ultimate failure of the rocks. Thus, this mechanism operates even in the development of extension fractures in thin beds. But because the stresses in such thin beds are mainly controlled by bedding plane traction the hydraulic fracture mechanism plays only a secondary role.

In the present study, the number of thin veins included among the measured fractures was sufficiently small to be negligible. The quantity of fluid required to give rise to even a thin hydrothermal vein is extremely large. Also, if the vein material is quartz the transporting fluids must cool to enable the silica to come out of solution. This means the fluids usually originate at greater depths than that of fracture. We therefore conclude that the fractures we studied mainly resulted from high fluid pressures which obtained within the beds prior to the development of the fractures (see Fyfe *et al.* 1978, chaps 10 & 11 for further discussion).

In the following argument we will only consider the hydraulic fracture mechanism in relation to extension fracture development (for conditions leading to hybrid extension/shear fracture, see Price 1977). However, one may readily infer how the subsequent argument may also be applied to the development of hybrid fractures.

Hydraulic fracture occurs when the fluid pressure (p) in the rock exceeds the least principal stress (S_3) by an amount equal to the tensile strength of the rock (T), that is

$$S_3 - p = T. \qquad (2)$$

From the concepts of 'fracture mechanics' it is known that

$$T = \frac{\pi . K_{IC}}{2 . C_0} \qquad (3)$$

where, as before, T is the tensile strength, C_0 is the half length of the fracture and K_{IC} is a material property known as the critical stress intensity factor. Thus, the tensile strength T depends upon the length of the fracture. From Equations 2 and 3 one may infer that for a given value of S_3, the magnitude of the fluid pressure (p_p) required to propagate a developing fracture is smaller than the fluid pressure (p_i) required for fracture initiation (when the fracture plane is incipient or negligibly small).

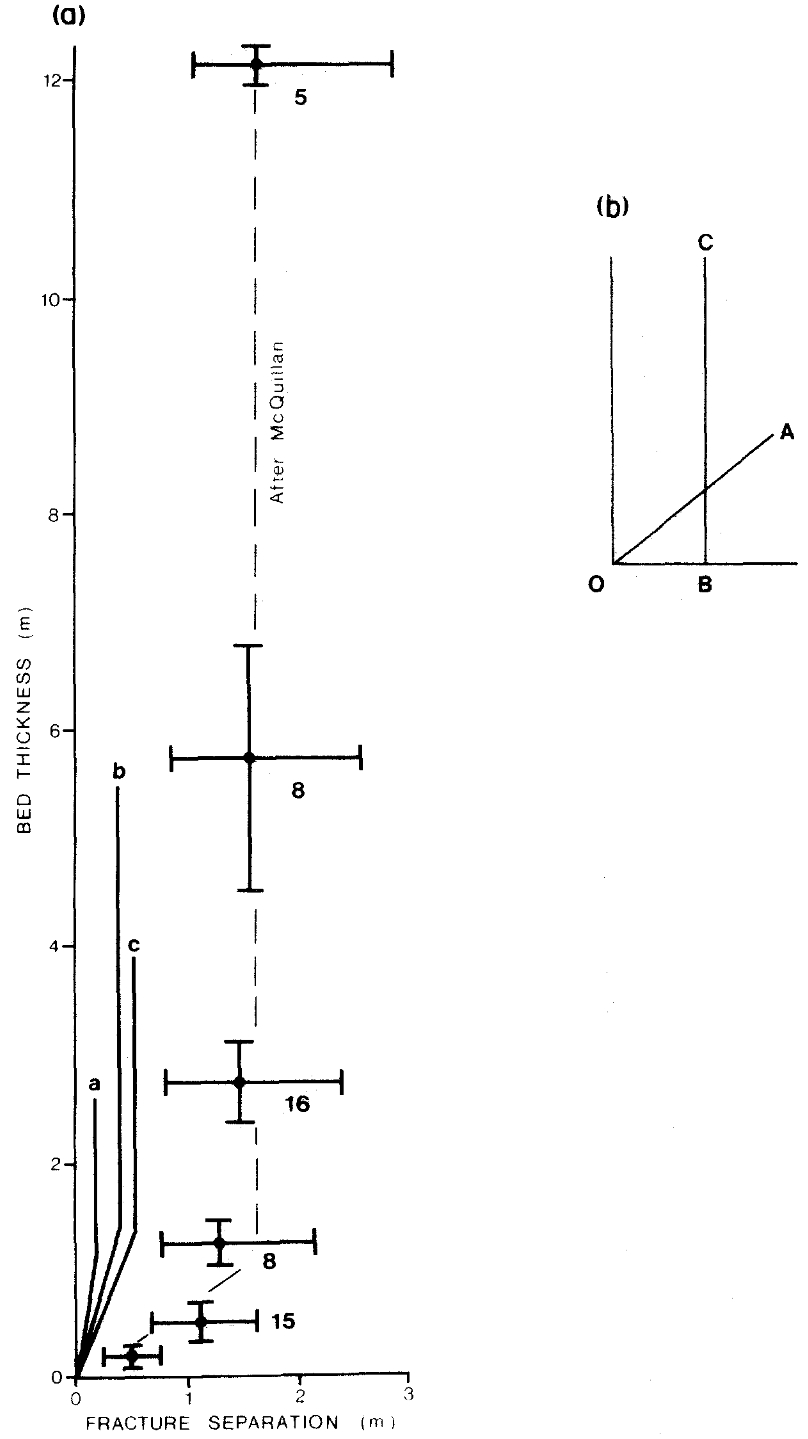

Fig. 3. (a) Curves a, b and c are the diagrammatic representation of data represented in Fig. 2. The remainder of the data are after McQuillan (1973), the numbers indicate the number of groups of readings. (b) See text for explanation.

Consequently, when the fracture is developing, there will be a fluid pressure gradient from the low pressure fluid (p_p) in the fracture to the higher pressure fluid (p_i) at some distance '*d*' into the unfractured rock, as indicated in Fig. 4. Therefore, at a distance '*d*' from the fracture the rock is 'unaware' of the existence of the fracture. Thus at this distance from the fracture the original conditions for hydraulic fracture are in existence and a second fracture could develop. Fracture separation would therefore be quite closely linked to the distance '*d*', which is clearly related to the gradient of the fluid pressure dp/dx, as indicated in Fig. 4. In turn, the fluid pressure gradient will largely be determined by (1) the rate of propagation of the fracture (which will be related to the strain rate and would therefore reflect the 'degree of deformation') and (2) the permeability ($\bar{K}$) of the unfractured rock (which, of course, would be related to the lithology). With reference to the second of these factors: although data are lacking, one may infer that fractures in greywacke beds are more closely spaced than those in the limestone. By the same

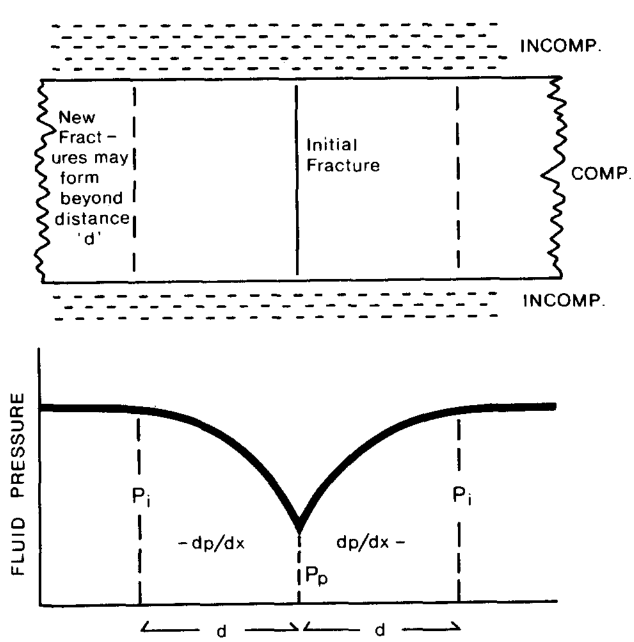

Fig. 4. Indicates how an initial fracture may disrupt the fluid pressure in its vicinity thereby preventing another fracture from developing except at some distance *d* (or greater) where the original fluid pressure (P_i) is undisturbed (see text for details).

token, one would expect fractures in thick, porous, coarse-grained sandstones (with low values of $\bar{K}$) to be yet more widely spaced.

From Figs. 1 and 2 it will be noted that the thickness of the adjacent incompetent layer has a slight influence on the spacing of fractures in the thicker beds. It would seem that traction effects may be invoked.

The arguments regarding mechanisms outlined above are qualitative and much more data are required before a rigorous and quantitative analysis can be attempted. However, no matter what our understanding, or lack of understanding, may be regarding mechanisms, we suggest that the empirical relationships arrived at are valid and have obvious importance in a number of practical applications which would include some problems in the petroleum industry, hydrology and nuclear waste disposal.

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